NNs Based Asymptotic Adaptive Observer without Riccati Constraint and Chattering

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Abstract— In this paper an adaptive observer for a class of uncertain nonlinear systems is proposed. Based on linearly parameterized neural networks, Lyapunov argument, and an adaptive bounding technique, the proposed scheme ensures zero observer error convergence, asymptotically, even in the presence of approximation error and disturbances, whereas the others error signals remain bounded. In addition, the proposed scheme does not rely on any Riccati equation solution and it does not suffer from chattering.

Keywords— Nonlinear systems, adaptive observers, neural networks.

1 Introduction

Recently in [1] and [2] it was considered the asymptotic observation, based on linearly parameterize neural networks (LPNNs), of a class of uncertain nonlinear systems in the presence of time-varying unknown parameters and non-vanishing disturbances. However, the proposed methods present several drawbacks, which restrict the application: 1) observer in [1] is based on a decaying-width design, hence it can exhibit chattering phenomenon when the width has decayed practically to zero, 2) observer in [2] is discontinuous, then also exhibit chattering, and it assume that the unknown parameters have absolutely integrable derivatives. In addition, observers in [1] and [2] rely on a Riccati equation solution to be implemented.

On the other hand, in Vargas and Hemerly [3], a robust modification for the weight adaptive law in neuro-identification problems was proposed to ensure, in contrast to the literature, that the prediction error converges to zero in the presence of approximation error and disturbances. The adaptive law consisted of a leakage modification of a standard gradient descent algorithm. However, in contrast to commonly leakage modifications [13] which aim at stability in the presence of approximation errors and disturbances, the leakage term was introduced for, in addition to stability, ensuring that the state error converges to zero. It was proved by using usual Lyapunov arguments and an adaptive bounding technique [14] that the state error converges asymptotically to zero, whereas the others error signals remain bounded. However, the proposed method relies on the complete state measurement. In fact, the main contributions of this seminal work were to provide an appealing parameterization and weight adaptive laws to asymptotical identification. So, asymptotic observation or tracking were not consider.

In this paper, motivated by the previous facts, the problem of estimating the state of an uncertain system is considered. The aim is to relax the application assumption in [1] and [2], that is, the Riccati equation constraint, whereas the chattering is avoided. Based on neural networks, Lyapunov method, an adaptive bounding technique, and by using the design methodology introduced in [3], an asymptotic adaptive observer is proposed for a more general class of unknown nonlinear systems that these in [1] and [2]. It is proved that the observation error converges asymptotically to zero, even in the presence of approximation errors and disturbances, if some conditions on the design parameters are provided. The proposed work extends the state of the art in adaptive observer design, since it ensures convergence of the observation error to zero, without chattering and Riccati constraint. To the best of our knowledge, a smooth adaptive observer which does not suffer from chattering and ensures convergence of the estimated state to the true has not been established in the literature yet. Throughout the paper $tr(\cdot)$ denotes the trace operator, $\lambda_{min}(\cdot)$ denotes the minimum eigenvalue operator, $\|\cdot\|$ denotes the 2-norm and $\|\cdot\|_F$ denotes the Frobenius norm.

2 Linearly parameterized neural networks

LPNNs can be expressed mathematically as

$$\rho_{nn}(W,\zeta)=W\pi(\zeta) \tag{1}$$

where $W\in\Re^{r\times L_\rho}$, $\zeta\in\Re^{L_\zeta}$, $\pi:\Re^{L_\zeta}\mapsto\Re^{L_\rho}$ is the so-called basis function vector, which can be considered as a nonlinear vector function whose arguments are preprocessed by a scalar function $s(\cdot)$, and r, L_ρ, L_ζ are integers strictly positive. Commonly used scalar functions $s(\cdot)$ include sigmoid, tanh, gaussian, Hardy's, inverse Hardy's multiquadratic, etc [4]. However, here we are only interested in the class of LPNNs for which $s(\cdot)$ is bounded, since in this case we have,

$$\|\pi(\zeta)\| \le \pi_0 \tag{2}$$

being π_0 a strictly positive constant.

The class of LPNNs considered in this work includes HONN [5], RBF networks [6], wavelet networks [7], and also others linearly parameterized approximators as Takagi-Sugeno fuzzy systems [8], which satisfy the so-called universal approximation property:

Property 1 [9]**:** Given a constant $\varepsilon_0 > 0$ and a continuous function $f : \Omega \mapsto \Re^r$, where $\Omega \subset \Re^{L_\zeta}$ is a compact set, there exists a weight matrix $W = W^*$ such that the output of the neural network architecture (where L_ρ may depend on ε_0 and f) satisfies

$$sup_{\zeta \in \Omega} \left| f(\zeta) - W^* S(\zeta) \right| \le \varepsilon_0 \tag{3}$$

where $|\cdot|$ denotes the absolute value if the argument is a scalar. If the argument is a vector function in $\Re^r$ then $|\cdot|$ denotes any norm in $\Re^r$.

3 Problem formulation

Consider the class of nonlinear systems

$$\begin{aligned} \dot{x} &= Ax + B[f(x,u) + h(x,u,v,t)], \quad x(0) = x_0 \\ y &= Cx \end{aligned} \tag{4}$$

where $x \in X$ is the n-dimensional state vector, $u \in U$ is a m-dimensional admissible input vector, $v \in V \subset \Re^q$ is a vector of time varying uncertain variables, $y \in Y$ is the q-dimensional output vector, h are internal or external disturbances, $f : X \times U \mapsto \Re^r$ and $h : X \times U \times V \times [0,\infty) \mapsto \Re^r$ are unknown continuous maps, A, B, C are known matrices of appropriate dimensions. In order to have a well-posed problem, we assume that X, U, V are compact sets, f and h are locally Lipschitzian with respect to x in $X \times U \times V \times [0,\infty)$, such that (4) has a unique solution.

We assume that the following can be established

Assumption 1: On a region $X \times U \times V \times [0,\infty)$

$$\|h(x,u,v,t)\| \le h_0 \tag{5}$$

where $\bar{h}_0$, such that $\bar{h}_0 > h_0 \ge 0$, is a known constant.

Assumption 2: The pair *A*, *C* is detectable and there exists a symmetric positive definite matrix *P* such that

$$P(A - LC) + (A - LC)^T P = -Q < 0 \tag{6}$$

$$B^T P = C^* \tag{7}$$

where C^* lies in the span of the rows of *C*.

Remark 1: Assumption 1 is common in approximation theory. Assumption 2 implies that the linear part of the unknown system is dissipative or strictly positive real [10].

The aim is to design a NNs-based adaptive observer for (4) to ensure the observation error convergence, that is, the state error convergence to zero, asymptotically, which will be accomplished despite the presence of approximation error and disturbances.

4 Neural parameterization and observer error equation

We start by presenting the observer model and the definition of the relevant errors associated with the problem.

From (3), by using LPNNs, the nonlinear mapping $f(x,u)$ can be replaced by $W^* \pi(x,u)$ plus an approximation error term $\varepsilon(x,u)$. More exactly, (4) becomes

$$\dot{x} = Ax + BW^* \pi(x,u) + B\varepsilon(x,u) + Bh(x,u,v,t) \tag{8}$$

where $W^* \in \Re^{rxL}$ is an "optimal" or ideal matrix, which can be defined as

$$W^* := \arg \min_{\hat{W} \in \Gamma} \left\{ \sup_{\substack{x \in X, \\ u \in U}} \left| f(x,u) - \hat{W}\pi(x,u) \right| \right\} \tag{9}$$

with $\Gamma = \left\{ \hat{W} \;\middle|\; \|\hat{W}\| \le \alpha_{\hat{W}} \right\}$, $\alpha_{\hat{W}}$ is a strictly positive constant, $\hat{W}$ is an estimate of W^*, and $\varepsilon(x,u)$ is an approximation error term, corresponding to W^*, which can be defined as

$$\varepsilon(x,u) := f(x,u) - W^* \pi(x,u) \tag{10}$$

The approximation, reconstruction, or modeling error ε is a quantity that arises due to the incapacity of LPNNs to match the unknown map $f(x,u)$.

Remark 2: It should be noted that W^* and $\varepsilon(x,u)$ might be nounique. However, $\|\varepsilon(x,u)\|$ is unique by (9).

The structure (8) suggests an observer of the form

$$\dot{\hat{x}} = A\hat{x} + B\hat{W}\pi(\hat{x},u) - L\tilde{y} \tag{11}$$

where $\hat{x}$ is the estimated state, $L \in \Re^{n\times n}$ is a positive definite feedback gain matrix introduced to attenuate the effect of the nonzero uncertainties and the initial condition x_0, and $\tilde{y} = C\hat{x} - y$ is the output error. It will be demonstrated that the observer (11) used in conjunction with a convenient adjustment law for $\hat{W}$, to be proposed in the next section, ensures the asymptotic convergence of the state error to zero, even in the presence of the approximation error and disturbances.

By defining the state estimation error as $\tilde{x} := \hat{x} - x$, from (8) and (11), we obtain the observer error equation

$$\begin{aligned}\dot{\tilde{x}} &= -L\tilde{x} + B\tilde{W}\pi(x,u) - B\varepsilon(x,u) - Bh(x,u,v,t) \\ &\quad + B\omega(\hat{x},x,u)\end{aligned} \tag{12}$$

where $\tilde{W} := \hat{W} - W^*$ is the error weight and $\omega(\hat{x},x,u) = \pi(\hat{x},u) - \pi(x,u)$ is a disturbance term.

Fact 1: With the definitions (2) and (9), the approximation error and disturbance terms are upper bounded by

$$\begin{aligned}&\|\varepsilon(x,u)\| \le \varepsilon_0 \\ &\|\omega(\hat{x},x,u)\| \le \omega_0\end{aligned} \tag{13}$$

where ε_0 and ω_0 are positive constants.

Assumption 3: The upper bounds $\bar{\varepsilon}_0 > \varepsilon_0$ and $\bar{\omega}_0 > \omega_0$ are previously known.

Remark 3: The previous knowledge of upper bounds for approximation error and disturbances is common in the robust on-line parameter estimation literature. For instance, the dead zone algorithm uses a previous knowledge of bounds for the approximation errors, as can be seen in [11]-[12], or modeling error, as reported in [13].

5 Adaptive laws and stability analysis

This section is concerned with the definition of the weight adaptive law to force the observer error to be null, asymptotically, whereas, at the same time, the others error signals remain bounded. The design method is similar to that in [3]. However, the adaptive laws are now defined based on the output error. In contrast to [3] where it was assumed that the state is completely measurable.

Before presenting the main theorem, we state a fact, remark and lemma, which will be used in the stability analysis.

Fact 2: Let $W^*, W_0, \hat{W}, \tilde{W} \in \Re^{r\times L_\rho}$ and $\bar{C} \in \Re^{r\times r}$ be a diagonal matrix such that $\bar{C}^T\bar{C} = C$, where $C = diag(c_i)$, $c_i > 0$. Then, with the definition of $\tilde{W} = \hat{W} - W^*$, the following equalities are true:

$$\begin{aligned}2tr\left[\tilde{W}^T C\left(\hat{W} - W_0\right)\right] &= \left\|\bar{C}\tilde{W}\right\|_F^2 \\ &+ \left\|\bar{C}\left(\hat{W} - W_0\right)\right\|_F^2 - \left\|\bar{C}\left(W^* - W_0\right)\right\|_F^2 \\ 2tr\left[\hat{W}^T W_0\right] &= \left\|\hat{W}\right\|_F^2 + \left\|W_0\right\|_F^2 - \left\|\hat{W} - W_0\right\|_F^2\end{aligned} \tag{14}$$

Remark 4: The first equality in (14) leads to the following inequality:

$$\begin{aligned}2tr\left[\tilde{W}^T C\left(\hat{W} - W_0\right)\right] &\ge c_{i\,min}\left\|\tilde{W}\right\|_F^2 \\ &+ c_{i\,min}\left\|\hat{W} - W_0\right\|_F^2 - c_{i\,max}\left\|W^* - W_0\right\|_F^2\end{aligned} \tag{15}$$

where $c_{i\,max} = max(c_i)$ and $c_{i\,min} = min(c_i)$.

Lemma 5.1: Let a scalar bounding function be given by

$$\begin{aligned}\dot{\hat{\psi}} &= -\gamma_\psi\left\|C^*\tilde{x}\right\| \\ &\cdot\left[2\alpha_1 l\left(\hat{\psi},\psi^*\right)\hat{\psi} - \alpha_2\left(\left\|\hat{W}\right\|_F^2 + \left\|W_0\right\|_F^2\right) - 2\alpha_1 l\left(\hat{\psi},\psi^*\right)\psi^*\right]\end{aligned} \tag{16}$$

where

$$l\left(\hat{\psi},\psi^*\right) = \frac{2l_0}{\hat{\psi} + \psi^*} \tag{17}$$

and $\gamma_\psi, l_0, \alpha_1, \alpha_2, \psi^* > 0$. Then, subject to the condition

$$\hat{\psi}(0) \ge \delta\psi^* \tag{18}$$

where $\delta = \dfrac{4\alpha_1 l_0 + \alpha_2\left\|W_0\right\|_F^2}{4\alpha_1 l_0}$, the bounding function

is lower bounded, for all $t \ge 0$, by

$$\hat{\psi}(t) \geq \delta\psi^* \tag{19}$$

Proof: Consider the Lyapunov-like function ([13])

$$V_\psi = \hat{\psi}\,\gamma_\psi^{-1}\hat{\psi}/2 \tag{20}$$

By taking the derivative of (20) along (16) we obtain

$$\dot{V}_\psi = -\hat{\psi}\left\|C^*\tilde{x}\right\| \cdot\left[2\alpha_1 l\hat{\psi} - \alpha_2\left(\left\|\hat{W}\right\|_F^2 + \left\|W_0\right\|_F^2\right) - 2\alpha_1 l\psi^*\right] \tag{21}$$

Furthermore, based on (16) and (18) it follows that $\hat{\psi}(t) > 0$ for all $t \geq 0$. Then, with the definition (17), the Lyapunov derivative (21) can lower bounded as

$$\dot{V}_\psi \geq -2\alpha_1 l\,\hat{\psi}\left\|C^*\tilde{x}\right\|\left[\hat{\psi} - \delta\psi^*\right] \tag{22}$$

Hence, if $\hat{\psi} \leq \delta\psi^*$ we have $\dot{V}_\psi \geq 0$, which implies that the bounding function is directed towards the outside or boundary of the region $\left\{\hat{\psi}\ \middle|\ \hat{\psi} \leq \delta\psi^*\right\}$. Consequently, based on (18), it follows that $\hat{\psi} \geq \delta\psi^*$ for all $t \geq 0$.

□

We now state and prove the main theorem of the paper.

Theorem 5.1**:** Consider the class of uncertain nonlinear systems described by (4), which satisfy Assumptions 1-3. Let the weight law be given by

$$\dot{\hat{W}} = -\gamma_W\left\{2C\left(\hat{\psi} - \psi^*\right)\left[\hat{W} - \left(I - \alpha_2 C^{-1}\right)W_0\right]\left\|C^*\tilde{x}\right\|\right\} + C^*\tilde{x}\pi^T(\hat{x},u)\Big\} \tag{23}$$

where $\hat{\psi}$ is given by (16), $\gamma_W > 0$, I is an identity matrix, and

$$K = P^T + P \tag{24}$$

Then, subject to the condition (18), and if

$$\psi^* = \frac{2\alpha_4\left\|KB\right\|_F}{\alpha_1 l_0} \tag{25}$$

$$\alpha_2 \leq c_{i\,min} \tag{26}$$

$$tr\left(W^{*T}W_0\right) \leq 0 \tag{27}$$

$$\beta_1 \leq \left\|W^* - W_0\right\|_F \leq \beta_2 \tag{28}$$

where

$$\alpha_4 = \bar{\varepsilon}_0 + \bar{h}_0 + \bar{\omega}_0, \qquad \beta_1 = \frac{4\alpha_1 l_0}{\left\|W_0\right\|_F\sqrt{\alpha_2 c_{i\,max}}}, \qquad \beta_2 = \sqrt{\frac{\alpha_1 l_0}{2c_{i\,max}}},\ \alpha_3 = \lambda_{min}(Q)\Big/\left\|C^*\right\| \tag{29}$$

The error signals $\tilde{x}, \tilde{W}, \tilde{\psi}$ are uniformly bounded and $lim_{t\to\infty}\tilde{x}(t) = 0$.

Proof: Consider the candidate Lyapunov function

$$V = \tilde{x}^T P\tilde{x} + tr\left(\tilde{W}^T\gamma_W^{-1}\tilde{W}\right)/2 + \tilde{\psi}\,\gamma_\psi^{-1}\tilde{\psi}/2 \tag{30}$$

where $\tilde{\psi} = \hat{\psi} - \psi^*$.

By evaluating (30) along the trajectories of (12), (16) and (23), and using the representation $tr\left(\tilde{W}^T C^*\tilde{x}\pi^T\right) = \tilde{x}^T C^{*T}\tilde{W}\pi$, we obtain

$$\begin{aligned}\dot{V} = &-\tilde{x}^T\left[(A-LC)^T P + P(A-LC)\right]\tilde{x} - \tilde{x}^T C^{*T}KB(\varepsilon + h - \omega)\\ &-2\tilde{\psi}\left\|C^*\tilde{x}\right\| tr\left[\tilde{W}^T C\left(\hat{W} - W_0\right)\right] - 2\alpha_2\tilde{\psi}\left\|C^*\tilde{x}\right\| tr\left(\tilde{W}^T W_0\right)\\ &-2\alpha_1 l\tilde{\psi}\hat{\psi}\left\|C^*\tilde{x}\right\| + \alpha_2\left(\left\|\hat{W}\right\|_F^2 + \left\|W_0\right\|_F^2\right)\tilde{\psi}\left\|C^*\tilde{x}\right\|\\ &+2\alpha_1 l\psi^*\tilde{\psi}\left\|C^*\tilde{x}\right\|\end{aligned} \tag{31}$$

By using Fact 2, the representation $2\tilde{\psi}\hat{\psi} = \tilde{\psi}^2 + \hat{\psi}^2 - \psi^{*2}$, and (6), the Lyapunov derivative can be written as

$$\begin{aligned}\dot{V} = &-\tilde{x}^T Q\,\tilde{x} - \tilde{x}^T C^* KB(\varepsilon + h - \omega)\\ &-\tilde{\psi}\left\|C^*\tilde{x}\right\|\left[\left\|\bar{C}\tilde{W}\right\|_F^2 + \left\|\bar{C}\left(\hat{W} - W_0\right)\right\|_F^2 - \left\|\bar{C}\left(W^* - W_0\right)\right\|_F^2\right]\\ &+2\alpha_2\tilde{\psi}\left\|C^*\tilde{x}\right\| tr\left(W^{*T}W_0\right) - \alpha_1 l\left(\tilde{\psi}^2 + \hat{\psi}^2 - \psi^{*2}\right)\left\|C^*\tilde{x}\right\|\\ &+\alpha_2\left\|\hat{W} - W_0\right\|_F^2\tilde{\psi}\left\|C^*\tilde{x}\right\| + 2\alpha_1 l\psi^*\tilde{\psi}\left\|C^*\tilde{x}\right\|\end{aligned} \tag{32}$$

Furthermore, by using Remark 4, condition (27), Lemma 5.1, and notation (29), the Lyapunov derivative (32) can upper bounded as

$$\begin{aligned}\dot{V} \leq \left\|C^*\tilde{x}\right\|\cdot\Big[&-\alpha_3\left\|\tilde{x}\right\| + \alpha_4\left\|KB\right\|_F\\ &-\tilde{\psi}\left(c_{i\,min}\left\|\tilde{W}\right\|_F^2 + c_{i\,min}\left\|\hat{W} - W_0\right\|_F^2 - c_{i\,max}\left\|W^* - W_0\right\|_F^2\right)\\ &-\alpha_1 l\left(\tilde{\psi}^2 + \hat{\psi}^2 - \psi^{*2}\right) + \alpha_2\left\|\hat{W} - W_0\right\|_F^2\tilde{\psi} + 2\alpha_1 l\psi^*\tilde{\psi}\Big]\end{aligned} \tag{33}$$

Further using (26) and rearranging terms, we obtain

$$\dot{V} \le \left\| C^* \tilde{x} \right\| \cdot \Big[-\alpha_3 \|\tilde{x}\| - c_{i\,min} \tilde{\psi} \left\| \tilde{W} \right\|_F^2 - \alpha_1 l \tilde{\psi}^2 + \alpha_4 \|KB\|_F + c_{i\,max} \tilde{\psi} \left\| W^* - W_0 \right\|_F^2 - \alpha_1 l \left(\hat{\psi}^2 - {\psi^*}^2 \right) + 2\alpha_1 l \psi^* \tilde{\psi} \Big] \quad (34)$$

By employ the definition of ψ^*, see (25), recalling that $\tilde{\psi} = \hat{\psi} - \psi^*$, and using Lemma 5.1, (34) reduces to

$$\dot{V} \le \left\| C^* \tilde{x} \right\| \cdot \Big[-\alpha_3 \|\tilde{x}\| - c_{i\,min} \tilde{\psi} \left\| \tilde{W} \right\|_F^2 - \alpha_1 l \tilde{\psi}^2 + \left(\alpha_1 l_0 / 2 + c_{i\,max} \left\| W^* - W_0 \right\|_F^2 \right) \hat{\psi} - c_{i\,max} \psi^* \left\| W^* - W_0 \right\|_F^2 - \alpha_1 l \hat{\psi}^2 + \alpha_1 l {\psi^*}^2 + 2\alpha_1 l \psi^* \tilde{\psi} \Big] \quad (35)$$

which, by using (17), implies

$$\dot{V} \le \left\| C^* \tilde{x} \right\| \cdot \Big[-\alpha_3 \|\tilde{x}\| - c_{i\,min} \tilde{\psi} \left\| \tilde{W} \right\|_F^2 + \left(\alpha_1 l_0 / 2 + c_{i\,max} \left\| W^* - W_0 \right\|_F^2 \right) \hat{\psi} - c_{i\,max} \psi^* \left\| W^* - W_0 \right\|_F^2 - \frac{2\alpha_1 l_0}{\hat{\psi} + \psi^*} \hat{\psi}^2 + 4\alpha_1 l_0 \psi^{*2} / \hat{\psi} \Big] \quad (36)$$

Thus by using Lemma 5.1 and rearranging terms in (36), we finally obtain

$$\dot{V} \le \left\| C^* \tilde{x} \right\| \cdot \Big\{ -\alpha_3 \|\tilde{x}\| - c_{i\,min} \tilde{\psi} \left\| \tilde{W} \right\|_F^2 - \psi^* \left(c_{i\,max} \left\| W^* - W_0 \right\|_F^2 - \frac{(4\alpha_1 l_0)^2}{\alpha_2 \|W_0\|_F^2} \right) - \frac{\alpha_1 \hat{\psi}^2}{\hat{\psi} + \psi^*} \left[l_0 - \left(l_0 / 2 + c_{i\,max} \left\| W^* - W_0 \right\|_F^2 / \alpha_1 \right) \right] - \frac{\alpha_1 \hat{\psi}}{\hat{\psi} + \psi^*} \left[l_0 \hat{\psi} - \left(l_0 / 2 + c_{i\,max} \left\| W^* - W_0 \right\|_F^2 / \alpha_1 \right) \psi^* \right] \Big\} \quad (37)$$

It addition, we note from (28) that

$$\left\| W^* - W_0 \right\|_F^2 \ge \frac{(4\alpha_1 l_0)^2}{\alpha_2 c_{i\,max} \|W_0\|_F^2}, \qquad \frac{l_0}{2} \ge c_{i\,max} \left\| W^* - W_0 \right\|_F^2 / \alpha_1 \quad (38)$$

By substituting (38) into (37), and using Lemma 5.1, we arrive at

$$\dot{V} \le -\lambda_{min}(Q) \|\tilde{x}\|^2 \quad (39)$$

Hence, the error signals $\tilde{x}, \tilde{W}, \tilde{\psi}$ are uniformly bounded. Further, since V is bounded from below and non increasing with time, we have

$$lim_{t \to \infty} \int_0^t \|\tilde{x}(\tau)\|^2 \, d\tau \le \frac{V(0) - V_\infty}{\lambda_{min}(Q)} < \infty \quad (40)$$

where $lim_{t \to \infty} V(t) = V_\infty < \infty$. Notice that with the bounds on $\tilde{x}, \tilde{W}, \tilde{\psi}, \varepsilon$ and h, $\|\tilde{x}\|^2$ is uniformly continuous. Thus from (12), it follows that $\dot{\tilde{x}}$ is bounded. Hence by Barbalat´s lemma [13], we conclude that $lim_{t \to \infty} \tilde{x}(t) = 0$.

□

Remark 5: It should be highlighted that $C^* \tilde{x}$ in (16) and (23) can be computed via $\tilde{y}$. Since C^* lies in the span of the rows of *C*, there exists a matriz *T* such that $C^* = TC$. In [2] it is shown a way to determine *T*: consider the singular value decomposition of C,

$$C = U \Sigma V^T \quad (41)$$

and the pseudo inverse C^+,

$$C^+ = V \Sigma^+ U^T \quad 42)$$

where $U \in \Re^{q \times q}$ e $V \in \Re^{n \times n}$ are orthogonal matrices, $\Sigma = \begin{bmatrix} D & 0_{k \times (n-k)} \\ 0_{(q-k) \times k} & 0_{(q-k) \times (n-k)} \end{bmatrix}$, $D = diag(\sigma_i, i = 1,2,..,k)$, $\sigma_i > 0$ are the singular values of *C*, *k* is the rank of *C*, and $\Sigma^+ = \begin{bmatrix} D^{-1} & 0_{k \times (q-k)} \\ 0_{(n-k) \times k} & 0_{(n-k) \times (q-k)} \end{bmatrix}$. Then, $T = C^* C^+$, since it satisfies the equation $C^* = TC$. In summary, $C^* \tilde{x} = TC\tilde{x} = T\tilde{y}$.

Remark 6: Conditions (6), (18), (24), and (26) are trivial since them are defined by the user according to a desired performance. Condition (25) implies the previous knowledge of upper bounds for the approximation error and disturbances, which is ensured by Assumption 1 and 3. Conditions (27) and (28) require at least that the sign of some entry of W^* and bounds for the ideal weights are known. The previous knowledge of bounds for the modeling error and ideal weights is not peculiar to the proposed scheme. Most robust modifications in the literature, as for example, switching-σ, parameter projection, and dead-zone require *a priori* information on the plant or modeling error for ensuring stability, as reported in [13].

Remark 7: It should be noted that condition (28) can be rewritten as

$$\frac{2c_{i\,max}}{\alpha_1}\left\|W^* - W_0\right\|^2 \leq l_0 \leq \frac{\sqrt{c_{i\,max}\alpha_2}\left\|W_0\right\|}{4\alpha_1}\left\|W^* - W_0\right\| \quad (43)$$

Hence, there is at least one way of selecting the design parameters to satisfy this interval condition: by selecting the design constant α_2 to be large enough and, in the sequence, by adjusting $\left\|W^* - W_0\right\|_F$ to be small enough, what can be achieved by appropriate selection of the matrix W_0 .

6 Conclusions

In this paper an adaptive observer that ensures observer error convergence to zero, even in the presence of approximation error and disturbances, was proposed. The main peculiarities of the proposed scheme are that it does not assume the existence of a solution for a Riccati equation, as usual in the literature, and it does not suffer from chattering phenomena. The use of the proposed methodology for implementing an asymptotic tracking control system is under investigation.

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References

[1] J. A. R. Vargas and E. M. Hemerly, "Observação adaptativa assintótica de sistemas Incertos," *Anais do XVI Congresso Brasileiro de Automatica*, pp. 1686-1691, Salvador, Oct. 2006.

[2] V. Stepanyan e N. Hovakimyan, "Robust adaptive observer design for uncertain systems with bounded disturbances," *Proc. of the 2005 IEEE Conference on Decision and Control, and the European Control Conference*, pp. 7750-7755, Sevilla-Spain, Dec. 2005.

[3] J. A. R. Vargas and E. M. Hemerly, "System neuro-identification with asymptotic convergence in the presence of approximation error and disturbances," submitted to *IEEE Transaction on Neural Networks*, 2007.

[4] S. S. Ge, C. C. Hang, T. H. Lee, and T. Zhang, *Stable adaptive neural network control*, Kluwer academic publishers, 2002.

[5] E. B. Kosmatopoulos, M. M. Polycarpou, M. A. Christodoulou, and P. A. Ioannou, "High-order neural network structures for identification of dynamical systems," *IEEE Trans. Neural Networks*, vol. 6, pp. 422-431, Mar. 1995.

[6] R. M. Sanner and J. J. E. Slotine, "Stable recursive identification using radial basis function networks," *Proc. 1992 ACC*, Chicago, IL, pp. 1829-1833, June 1992.

[7] Q. Zhang and A. Benveniste, "Wavelet networks," *IEEE Trans. Neural Networks*, vol. 3, pp. 889-898, 1992.

[8] L. X. Wang, *Adaptive fuzzy system and control: design and stability analysis*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1994.

[9] M. M. Polycarpou and P. A. Ioannou, "Identification and control of nonlinear systems using neural network models: design and stability analysis," Tech. Rep. 91-09-01, Dept. Elect. Eng. Syst., Univ. Southern Calif., Los Angeles, 1991.

[10] R. Rajamani, R., "Observers for Lipschitz nonlinear systems," *IEEE Trans. Automatic Control*, vol. 43, no. 3, pp. 397-401, 1998.

[11] Q. Song, "Robust training algorithms of multilayered neural networks for identification of nonlinear dynamical systems," *IEE Proc.- Control Theory Appl.*, vol. 145, no. 1, pp. 41-46, 1998.

[12] S. Jagannathan and F. L. Lewis, "Identification on nonlinear dynamical systems using multilayered neural networks," *Automatica*, vol. 32, no. 12, pp. 1707-1712, 1996.

[13] P.A. Ioannou and J. Sun, *Robust adaptive control*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1996.

[14] M. M., Polycarpou, "Stable adaptive neural control scheme for nonlinear systems," *IEEE Trans. Automatic Control*, vol. 41, no. 3, pp. 447-451.